Peri Tōn Mathēmatōn. (Apeiron, volume XXIX no. 4). Edited by Ian Mueller. Edmonton, Alberta, Canada: Academic Printing and Publishing, 1991. xii+251 pages. Index by name and subject. Index of passages cited.

Published in *Historia Mathematica* **22** (1995), 84–87.

The essays contained in the present volume are intended to "illustrate both the richness of Greek mathematical science itself and the variety and extent of its impact on Ancient Greek and subsequent culture" (p. vii). With only seven essays, each of which treats a very specific theme, "illustration" is indeed the maximum that can be asked for.

The first essay (Benno Artmann, "Euclid's *Elements* and its Prehistory", pp. 1–47) starts by an overview of the contents of this key work. This is followed by a chapter on its prehistory, applying what Proclus tells regarding earlier authors and the types of earlier *Elements* (shorter and longer proofs, avoidance of indirect proofs or of proportions) to the Euclidean work, in part with reference to the analyses of Oskar Becker, Erwin Neuenschwander, and others. Finally, the question of the application of areas and of "geometric algebra" is reviewed.

When retelling what Euclid has done, Artmann balances between (i) "the principle that we must not ascribe to thinkers, especially those of earlier times, 'either the principles of their consequences or the consequences of their principles'" (p. 2), and (ii) the idea that the only alternative to the distinctive attitude of mathematicians – that "the content of a proposition" can be separated "from its form of expression" – is the habit of philologists to "stress the particularity of different forms of expression" (p. 47). Though absent from the latter enunciation of principles, however, hermeneutic analysis of meanings rather than forms of expression is certainly present in the actual inquiry.

Andrew Barker's presentation of "Three Approaches to the canonic Division" (pp. 49–83) starts out by a presentation of the monochord and the various attunements as listed by Aristoxenus, the antimathematician *par excellence* of ancient musicology. On this background, three mathematical approaches to the division of the double octavo are discussed: the Euclidean *Sectio canonis* together with a variant ascribed to Thrasyllus; the approach of Plato (in *Timaeus*) and Archytas through means and proportions; and finally Ptolemy's explanation in terms of melodic intervals determined from epimoric ratios.

By necessity, the presentation of the terse Euclidean description concentrates on its actual determination of the division points and on the weak points and contradictions of the outcome. When discussing the other approaches, Barker takes up the question of motivations and aims – thus Plato's and Archytas's search for a mathematical theory *explaining* either what is truly harmonic irrespective of actual musical practice (Plato) or the harmonies of actual attunements, and Ptolemy's wholly innovative solution to this problem not by mathematical *fiat* but by way of connecting the problem of mathematical intelligibility to a theory of perception.

Ian Mueller has written on "Mathematics and Education: Some Notes on the Platonic

Program" (pp. 85–104). The program referred to is evidently that of *Republic* VII, but other parts of the Platonic corpus are drawn upon, both for elucidation of the *Republic* scheme and for other purposes. Of particular interest is a thorough discussion of the distinction between *arithmetic* and *logistic*, where strong arguments are set forth against the widespread interpretation – going back to Geminus – of the difference as being that between the pure and the applied study of number. This difference, as shown by Mueller, applies to each of the two domains. Instead, it is argued, the distinction between arithmetic and logistic that Plato does make in some places is the distinction between *counting* and *calculating*, in agreement with the etymologies of the two terms. Three sections on the discussions of incommensurability in *Laws* 819–20 and *Theaetetus* 147d–148b and on the curriculum proposed in *Epinomis* (most probably by Philip of Opus, mathematician and follower of Plato) suggest, among other things, that Plato was particularly impressed by what the young Theatetus had done because the introduction of "commensurability with regard to dýnamis" could be seen as a promise that number might be made applicable in "cases in which there is initially no indication that it will be applicable" (p. 98); further, that "Plato was unclear about the difference between asserting and defining" (p. 97); and that the fourth-century Academy was not worried by the methodological distinction between problems that were and problems that were not soluble using ruler and compass alone (p. 101f).

Edward Hussey's "Aristotle on Mathematical Objects" (pp. 105–133) analyses what may be meant by the elliptic discussion of these objects in *Metaphysics* M 1–3 – elliptic because Aristotle argues "recalling by compressed allusions thoughts familiar to himself and his intended audience" (p. 106). Obviously they are taken to *exist* somehow, but neither «apart from» sensible objects, nor as sensible objects themselves, to which they are prior in definition; if abstraction is understood as "the logical splitting up of a definition into its component parts", they may be called "abstracts" (p. 107).

Hussey suggests that Aristotle understood mathematical objects as "representative objects", identified with what the logician Kit Fine calls "arbitrary objects", of which "it is true [...] by definition that they possess just those properties which (i) are shared by all (actual or possible) individual members of the class they represent, and (ii) are *representative* properties, i.e., belong to the individuals qua members of that class" (p. 122) – we might perhaps speak of "objective abstracts" or "embodiments of distinctive properties". The interpretation seems adequate: on one hand because Aristotle's ontology operates with *potential* existence (evidently the only kind of existence that such object may possess); on the other, one might add, because the ontology presupposes the number of properties and categories to be exhaustible.

The essay closes with a discussion whether such objects are seen by Aristotle as *necessary to mathematics* or just as convenient devices (as is the fallacy of seeing them as separate from the sensible – convenient and innocuous according to Aristotle, but still a fallacy); it is concluded that they are at least the *proper objects* of mathematics (p. 129). In view of the conclusion (p. 133), this should exhaust the matter: "Aristotle on mathematics is altogether more sensible, more down-to-earth, and less liable to be carried away on logical hobby horses, than Plato or, indeed, some of his own subsequent interpreters. And when he writes about what mathematicians do, he writes about what they really do do, not what he thinks they ought to do".

Christian Marinus Taisbak offers "Elements of Euclid's *Data*" (pp. 135–171). Here as elsewhere, Taisbak the Socratic often speaks tongue in cheek; moreover, the analysis of the text is often done in counter-order, and the whole first part (until theorem 23) is omitted "since it does not deal with positions" (p. 157). All in all it may therefore be difficult to come to grips with the argument, not to speak of weighing the evidence.

Certain conclusions, however, stand out as important and well-established. Thus: that the *Data* is not meant to describe the construction of the entities whose derived givenness is asserted by the theorems, nor to prove the existence or uniqueness of solutions. Besides: that "givenness in position" should be understood in contrast to "moving" (potentially moving, it should perhaps be specified).

In the opinion of the present reviewer, on the other hand, the working hypothesis enunciated on p. 137 – that Euclid is trying to *axiomatize* the meaning of "being given" – is too strong. He is certainly trying to rationalize the domain, to establish connections and to draw conclusions; yet such rationalization is *a precondition*, a necessary step before axiomatization can be undertaken; but it is appraised unjustly if we mistake it for an attempt to axiomatize (under the circumstances by necessity an inept attempt, whether we take the *Elements* or the *Analytica posteriora* as the model for this process).

An important actor on the scene is the anonymous "Helping Hand" who sees that lines be drawn, points be taken, etc. (p. 144). In the *Data* as well as the *Elements*, indeed, all such constructions are asked for in the perfect imperative passive; they are never made by an "I", a "we" or a "you". Beyond the uses made of this observation by Taisbak in the analysis (uses that concern the conclusions already mentioned), this usage illustrates why Greek mathematics could never, as later done by Abū'l-Wafā' and other Muslim geometers, take up the theoretical problems of practical construction (how to certify the straightness of rulers, etc.) in a way that was relevant for practitioners. One might claim that no example ever discussed by Benjamin Farrington epitomizes so well the hegemony of slave holders' ideology even in domains where it could have no direct impact. Quite to the point, Taisbak characterizes his Helping Hand as "the well-known factotum in Greek geometry".

The last two essays step outside the Greek and into the Medieval Islamic orbit. Roshdi Rashed ("Archimède dans les Mathématiques Arabes", pp. 173–193) concentrates on the impact of Archimedes' infinitesimal methods. Only two Archimedean works on this topic were translated during the Golden Age (both twice in the ninth century): the *Measurement of the Circle* and *The Sphere and the Cylinder*. All the more remarkable is their influence, and the extent to which mathematicians from Thābit ibn Qurrah to ibn al-Haytham were able to go on from this starting point, in part reconstructing the methods used and the results found in other Archimedean works, in part going beyond these.

Rashed characterizes this tradition as neo-Archimedean. In one important respect, indeed, the participants did not only go beyond Archimedes' actual methods but also beyond their framework, making use both of what Rashed characterizes as affine transformations and of sophisticated arithmetical theory.

Len Berggren's essay on "Greek and Islamic Elements in Arabic Mathematics" (pp.

195–217) takes up this relation between Greek inspiration and the proper contribution of the Islamic world, largely by reviewing recent publications on the topics involved. In agreement with A. I. Sabra, Berggren emphasizes that the “reception” of Greek science was indeed a very active and selective process, and no mere passive reception of what was accidentally at hand. Similarly with Sabra, Berggren refers to a four-stage process: Acquisition of Greek science and philosophy; emergence of a large number of powerful Muslim thinkers with a thoroughgoing commitment to Hellenistic views; the stage of “ Islamization, in which [...] the mathematician is replaced by the expert in the arithmetic of inheritance law and the astronomer by the astronomical timekeeper in the mosque”; and finally the phase of sharp decline (p. 198). It is emphasized, however, that the process was not uniform, and that mathematicians with Hellenistic allegiance can be found in certain places until the very threshold of the modernization period.

All in all, as can be seen, the book covers the intended subject in breadth and – as could be expected from the list of authors – with clarity and in pointwise depth. Mathematical arguments are occasionally disturbed by misprints, but on the whole the book makes pleasant reading.

Jens Høyrup